

# From RHIC to LHC: A relativistic diffusion approach

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We investigate the energy dependence of stopping and hadron production in high-energy heavy-ion collisions based on a three-sources Relativistic Diffusion Model. The transport coefficients are extrapolated from Au + Au and Cu + Cu at RHIC energies ( $\sqrt{s_{NN}}=19.6$  - 200 GeV) to Pb + Pb at LHC energies  $\sqrt{s_{NN}}= 5.52$  TeV. Rapidity distributions for net protons, and pseudorapidity spectra for produced charged particles in central collisions are compared to data at RHIC energies, and discussed for several extrapolations to LHC energies.

**Key words** Relativistic heavy-ion collisions, Relativistic Diffusion Model, LHC predictions.

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# 1 Introduction

Stopping and particle production in relativistic heavy-ion collisions at the highest energies available at RHIC and LHC offer sensitive tools to test the nonequilibrium-statistical properties of these systems. Analytically soluble nonequilibrium-statistical models [1] not only allow to accurately describe a fairly large amount of phenomena, but also to predict results across the gap in center-of-mass energy from the highest RHIC energy in Au + Au collisions of  $\sqrt{s_{NN}}=200$  GeV [2, 3] to the LHC energy of  $\sqrt{s_{NN}}= 5.52$  TeV in Pb + Pb collisions.

Net-baryon (more precisely, net-proton) distributions have proven to be sensitive indicators for local equilibration, collective expansion, and deconfinement in heavy relativistic systems [4]. This is reconsidered in the analysis of SPS- and RHIC-results within the Relativistic Diffusion Model (RDM) for three sources in Sect. 2. In this work, we use the dependence of the transport coefficients on the center-of-mass energy from AGS via SPS to RHIC for predictions of net-baryon rapidity distributions at LHC, Sect. 3.

The model underlines the nonequilibrium-statistical features of relativistic heavy-ion collisions, but it also encompasses kinetic (thermal) equilibrium of the system for times that are sufficiently larger than the relaxation times of the relevant variables.

It is of particular interest in relativistic collisions of heavy systems to determine the fraction of produced particles that attains - or comes very close to - local statistical equilibrium. In the three-sources RDM, these are the particles produced in the midrapidity source. Hence we analyze Au + Au and Cu + Cu pseudorapidity distributions of produced particles at RHIC energies from  $\sqrt{s_{NN}}=19.6$  - 200 GeV. We determine the transport coefficients and numbers of produced particles as functions of the incident energies in Sect. 4.

For several reasonably motivated extrapolations of the transport coefficients we then calculate and discuss a range of resulting distribution functions for produced charged hadrons in Pb + Pb collisions at LHC energies. The conclusions are drawn in Sect. 5.

## 2 Net-baryon rapidity spectra

In the Relativistic Diffusion Model, the net-baryon rapidity distribution at RHIC energies emerges from a superposition of the beam-like nonequilibrium components that are broadened in rapidity space through diffusion due to soft (hadronic, low  $p_\perp$ ) collisions and particle creations, and a near-equilibrium (thermal) component at midrapidity that arises - among other processes - from hard (partonic, high  $p_\perp$ ) processes, and may indicate local quark-gluon plasma (QGP) formation.

The time evolution of the distribution functions is governed by a Fokker-Planck equation (FPE) in rapidity space [1, 4, 5, 6, 7]

$$\frac{\partial}{\partial t}[R(y, t)]^\mu = -\frac{\partial}{\partial y}[J(y)[R(y, t)]^\mu] + D_y \frac{\partial^2}{\partial y^2}[R(y, t)]^\nu \quad (1)$$

with the rapidity  $y = 0.5 \cdot \ln((E + p)/(E - p))$ . The rapidity diffusion coefficient  $D_y$  that contains the microscopic physics accounts for the broadening of the rapidity distributions due to interactions and particle creations, and it is related to the drift term  $J(y)$  by means of a dissipation-fluctuation theorem (Einstein relation) which is used to actually calculate  $D_y$  in the weak-coupling limit [1]. The drift  $J(y)$  determines the shift of the mean rapidities towards the central value, and linear and nonlinear forms have been discussed.

Here we use  $\mu = 1$  (due to norm conservation) and  $q = 2 - \nu = 1$  corresponding to the standard FPE, and a linear drift function

$$J(y) = (y_{eq} - y)/\tau_y \quad (2)$$

with the rapidity relaxation time  $\tau_y$ , and the equilibrium value  $y_{eq}$  of the rapidity [1, 16]. This is the so-called Uhlenbeck-Ornstein [8] process, applied to the relativistic invariant rapidity for the three components  $R_k(y, t)$  ( $k=1,2,3$ ) of the distribution function in rapidity space [1, 4, 7]

$$\frac{\partial}{\partial t}R_k(y, t) = \frac{1}{\tau_y} \frac{\partial}{\partial y}[(y - y_{eq}) \cdot R_k(y, t)] + D_y^k \frac{\partial^2}{\partial y^2}R_k(y, t). \quad (3)$$

Since the equation is linear, a superposition of the distribution functions [1, 4] emerging from  $R_{1,2}(y, t = 0) = \delta(y \mp y_b)$  and  $R_3(y, t = 0) = \delta(y - y_{eq})$  with mean values and variances obtained analytically from the moments equations yields the exact solution. With two sources and  $D_y$  calculated from the dissipation-fluctuation theorem, net-baryon rapidity spectra at low SIS-energies (about 1 GeV per particle) are well reproduced, whereas at AGS and SPS energies one finds discrepancies [1] to the data that rise strongly with  $\sqrt{s}$  due to the collective longitudinal expansion of the system.

With an effective diffusion coefficient  $D_y^{eff}$  that includes not only the random behaviour of the particles, but also the effect of expansion, however, the available net-proton data for central Au+Au collisions at AGS, and Pb+Pb at SPS [9] can be reproduced precisely with only two sources in the RDM. The corresponding rapidity width coefficient

$$\Gamma_y^{eff} = [8 \cdot \ln(2) \cdot D_y^{eff} \cdot \tau_y]^{1/2} \quad (4)$$

is shown in Fig.1 (middle frame) as function of the center-of-mass energy. The quotient of interaction and relaxation time  $\tau_{int}/\tau_y$  (Fig.1, upper frame) is determined from the peak positions of the data, and the corresponding longitudinal expansion velocity (Fig.1, lower frame) is obtained as in [10]. The rapidity distributions are shown in Fig. 2.

Extending the model to RHIC energies of 200 GeV, one finds [4] that within the linear approach with  $q = 1$  and two sources, it is impossible to reproduce the BRAHMS net-proton data [2] due to the high midrapidity yield.

This has recently been confirmed in an independent calculation by Alberico et al. [11] that uses the nonlinear approach, but also investigates the linear case. It has therefore been proposed in [4] that an expanding midrapidity source emerges. With this conjecture, the RHIC data can be interpreted rather precisely in the linear  $q = 1$  framework. A fraction of  $Z_{eq} \simeq 22$  net protons (55 net baryons) near midrapidity reaches local statistical equilibrium in the longitudinal degrees of freedom, Fig. 2. The variance of the equilibrium distribution  $R_{eq}(y)$  at midrapidity is broadened as compared to the Boltzmann result due to collective (multiparticle) effects. This corresponds to a longitudinal expansion (longitudinal flow) velocity of the locally equilibrated subsystem as in hydrodynamical descriptions. Here we obtain the expansion velocity as proposed in [10]

$$v_{coll}^{\parallel} = \left[ 1 - \left[ \frac{m_p}{m_p + \frac{1}{2} \cdot (T_{eff} - T)} \right]^2 \right]^{1/2} \quad (5)$$

with the limiting cases  $v_{coll}^{\parallel} = 1$  for  $T_{eff} \gg T$ , and  $v_{coll}^{\parallel} = 0$  for  $T_{eff} = T$ . The kinetic freezeout temperature is  $T \simeq 110$  MeV at RHIC energies, and  $T_{eff}$  is the effective temperature required to obtain the correct width of the midrapidity plateau. The proton mass is  $m_p$ .

The enhanced width of the midrapidity valley may also be accounted for in a non-extensive framework with two sources, through a value of  $q = 1.485$  [11, 12]. We believe, however, that a value of  $q$  larger than one covers the physical effect of longitudinal expansion which can be determined explicitly based on ordinary Boltzmann statistics.

Microscopically, the baryon transport over 4-5 units of rapidity to the equilibrated midrapidity region requires processes such as the nonperturbative gluon junction mechanism [13] to produce the observed central valley.

Macroscopically, the complete solution of (3) in the  $q = 1$  case is a linear superposition of nonequilibrium and local equilibrium distributions. The net-baryon rapidity distribution becomes

$$\frac{dN(y, t = \tau_{int})}{dy} = N_1 R_1(y, \tau_{int}) + N_2 R_2(y, \tau_{int}) + N_{eq} R_{eq}^{loc}(y) \quad (6)$$

with the interaction time  $\tau_{int}$  (total integration time of the differential equation). The number of net baryons (here: net protons) in local equilibrium near midrapidity is  $N_{eq}$ , and  $N_1 + N_2 + N_{eq}$  is equal to the total number of net baryons (corresponding to 158 net protons for central Au+Au). This yields a good representation of the BRAHMS data [2], as was already emphasized in [10]. Here the integration is stopped at the value of  $\tau_{int}/\tau_y$  that yields the best agreement with the data.

### 3 From RHIC to LHC energies

Based on the results for net protons from AGS to RHIC energies, we have extrapolated the diffusion-model parameters as functions of the center-of-mass energy to LHC energies, Fig. 1. To obtain the net-proton distributions at LHC energies of  $\sqrt{s_{NN}} = 14Z/A$  TeV  $\simeq 5.52$  TeV we use the following extrapolations for the time parameter and the effective

width coefficient with the logarithm of  $\sqrt{s_{NN}}$ :

$$\frac{\tau_{int}}{\tau_y}(\sqrt{s_{NN}}) = 2\exp(-1.1 \log(\sqrt{s_{NN}})) + 0.11 \quad (7)$$

$$\Gamma_y^{eff}(\sqrt{s_{NN}}) = 1.8 \log(\sqrt{s_{NN}}) + 0.2. \quad (8)$$

This allows to obtain a first approximation of the net-proton distribution at LHC energies as shown in Fig. 2. Here, the solid curve is for a midrapidity source with a particle content of 14% as at the highest RHIC energy, whereas the dashed curve corresponds to a particle content of only 7%.

Due to the small value of  $\tau_{int}/\tau_y = 0.14$  that is obtained from the extrapolation, the distribution functions at LHC energies extend well beyond the initial beam rapidity shown by an arrow. Hence, this result will have to be corrected at large rapidities since it is likely to violate kinematic constraints.

It should be mentioned that the dependence of the time parameter  $\tau_{int}/\tau_y$  on  $\sqrt{s_{NN}}$  can not be expected to be purely exponential over the whole energy scale. In particular, the curve is known to level off at low energies, as is evident from an earlier investigation of symmetric systems at SIS energies around  $\sqrt{s_{NN}} \simeq 2$  GeV [1]. Such a behaviour is plausible from the relation between the drift in rapidity space, and the energy loss that is associated with it. As has been pointed out by Koch [14], the mean rapidity loss per step can be written as

$$\langle \Delta y \rangle \propto \frac{y N_{part}}{\sinh(y)} \propto \frac{\tau_{int}}{\tau_y}. \quad (9)$$

This result is based on the usual relation between energy and rapidity

$$E = m \cosh(y), \quad (10)$$

or differentially

$$dE = m \sinh(y) dy. \quad (11)$$

On the other hand, the differential energy loss is proportional to the particle mass, the number of participants, and the rapidity

$$dE \propto m y N_{part} \quad (12)$$

such that the above Eq.(9) results. The expression is shown as a dashed curve in the upper frame of Fig.1. It levels off at small energies below 2 GeV ( $\tau_{int}/\tau_y = 1.98$  at 1

GeV) as a consequence of the relation between energy- and rapidity loss for net baryons. At large energies, the result of (9) shows a steeper fall-off which is difficult to reconcile with the data at the highest RHIC energy. For the extrapolation to LHC energies, we therefore use the exponential form in the net-proton case.

The fall of  $\tau_{int}/\tau_y$  with rising  $\sqrt{s_{NN}}$  does not imply that the lifetime of the locally equilibrated region becomes shorter with rising energy. According to common belief, the QGP-lifetime is expected to increase as the energy rises from RHIC to LHC.

## 4 Produced charged hadrons in the RDM

The Relativistic Diffusion Model [1] in its linear form with explicit treatment of the collective expansion [4], or in its nonlinear version with implicit consideration of the collective effects, is also suitable for the description and prediction of rapidity distributions of produced charged hadrons. Although the model results are somewhat more ambiguous because the initial conditions are not sharply defined as in case of the net-proton distributions, we proceed with the calculation of distribution functions for produced charged hadrons at RHIC and LHC energies.

If particle identification is not available, one has to convert the results to pseudorapidity space,  $\eta = -\ln[\tan(\theta/2)]$  with the scattering angle  $\theta$ . The conversion from  $y$ - to  $\eta$ - space of the rapidity density

$$\frac{dN}{d\eta} = \frac{dN}{dy} \frac{dy}{d\eta} = \frac{p}{E} \frac{dN}{dy} = J(\eta, \langle m \rangle / \langle p_T \rangle) \frac{dN}{dy} \quad (13)$$

is performed through the Jacobian

$$J(\eta, \langle m \rangle / \langle p_T \rangle) = \cosh(\eta) \cdot [1 + (\langle m \rangle / \langle p_T \rangle)^2 + \sinh^2(\eta)]^{-1/2}. \quad (14)$$

We approximate the average mass  $\langle m \rangle$  of produced charged hadrons in the central region by the pion mass  $m_\pi$ , and use a mean transverse momentum  $\langle p_T \rangle = 0.4$  GeV/c.

In the linear two-sources version, the RDM had been applied to pseudorapidity distributions of produced charged hadrons in Au+Au collisions at RHIC energies of 130 GeV and 200 GeV by Biyajima et al. [7]. Although the results were satisfactory, it soon turned out from the above net-proton results [4], and from general considerations, that an additional midrapidity source is required [15, 16].

Asymmetric relativistic systems are particularly sensitive to the details of the diffusion-model calculation, as was shown recently in our description of the d + Au system at 200 GeV in the three-sources model [16]. Here, an accurate modelling of the gradual approach of the system to thermal equilibrium was obtained. In particular, the dependence of the pseudorapidity distribution functions on centrality was precisely described. In the present investigation, however, we concentrate on symmetric systems because these will be of main interest at LHC. Our focus is on central collisions.

To allow for an extrapolation of our results for symmetric systems to LHC energies, we first perform RDM-calculations for Cu + Cu and Au + Au collisions at RHIC energies from 19.6 GeV via 62.4 GeV, 130 GeV to 200 GeV [22].

Typical results for central collisions of Au + Au at three energies are shown in Fig. 3 compared with PHOBOS data [3], and of Cu + Cu at two energies in Fig. 4 compared with preliminary PHOBOS data [23]. At the lowest energy, only two sources are needed for the optimization of the RDM-parameters in a  $\chi^2$ -fit, whereas three sources are indeed required at the higher energies. The particle number in the midrapidity source is indicated in the figures. At the highest energy of 200 GeV, the Cu + Cu system requires a smaller percentage of particles in the midrapidity source compared to Au + Au. This is consistent with the assumption that heavier systems are more likely to produce a locally equilibrated quark-gluon plasma.

The  $\chi^2$ -minimization program has been written in Mathematica for the purpose of this work [22]. In a previous investigation [16], the CERN minuit code [17] was used. We have verified that for 200 GeV Au + Au, the results are identical. When the execution stops at minimum  $\chi^2$ , the values of the time parameter, and of the effective widths of the partial distribution functions are determined.

The parameters of these calculations are summarized in Table 1. The number of particles produced in the three sources, the time parameters and the effective widths of the partial distributions (including the time evolution) are shown together with  $\chi^2/d.o.f.$ . The number of degrees of freedom (*d.o.f.*) is the number of data points minus the number of free parameters.

For produced charged hadrons, the results for the time parameter  $\tau_{int}/\tau_y$  as function of energy (Fig. 5, upper frame) are found to be significantly larger than the corresponding values for net protons (Fig.1). We have extrapolated the time parameter to LHC energies.



The upper curve in Fig. 5 has the functional dependence on energy that we have discussed for net baryons

$$\frac{\tau_{int}}{\tau_y} \propto \frac{yN_{part}}{\sinh(y)} \quad (15)$$

whereas the lower curve assumes an exponential dependence.

The resulting partial widths as functions of energy within the RHIC range for Au + Au are shown in the middle frames of Fig. 5 for both peripheral and midrapidity sources, which are not assumed to be equal for produced hadrons. As for net protons, the widths are effective values: they include the effect of collective expansion. We use log-extrapolations of the widths to LHC energies which yield the values given in the figures. Here we have plotted the values resulting from the  $\chi^2$ -minimization that include the time evolution up to  $\tau_{int}$

$$\Gamma_{1,2,eq}^{eff} = [8 \ln(2) \cdot D_{1,2,eq}^{eff} \cdot \tau_y \cdot (1 - \exp(-2\tau_{int}/\tau_y))]^{1/2} \quad (16)$$

The total number of produced charged particles relative to the number of participants resulting from the  $\chi^2$ -minimizations in various systems that we have investigated in the course of this work (including the asymmetric d + Au case) is shown in Fig. 6 as function of the center-of-mass energy. Extrapolating to LHC energies, we obtain 26.5 produced charged hadrons per participant pair in central collisions (0-6%). With an average number of 359.3 participants in the most central bin for Pb + Pb, this yields a total of 9520 produced charged hadrons.

In the Jacobian (14), the mean transverse momentum  $\langle p_T \rangle$  becomes significantly larger at LHC energies. Due to the increasing number of produced hadrons, the mean mass - which tends to approach the pion mass - decreases, such that  $(\langle m \rangle / \langle p_T \rangle)^2 \ll 1$  with increasing  $\sqrt{s_{NN}}$ , and the Jacobian

$$\frac{dy}{d\eta} = \frac{\cosh(\eta)}{\sqrt{1 + \frac{\langle m \rangle^2}{\langle p_t \rangle^2} + \sinh^2(\eta)}} \approx \frac{\cosh(\eta)}{\sqrt{1 + \sinh^2(\eta)}} = 1 \quad (17)$$

becomes very close to one at sufficiently high energy. Hence we use  $dy/d\eta = 1$  in our predictions of pseudorapidity distributions at LHC energies.

In view of the uncertainties that arise from the large energy gap between RHIC and LHC, we have tested variations of the diffusion-model parameters that indicate the spreading of our diffusion-model results within reasonable limits. Based on the extrapolations

of the time parameter and the effective widths to LHC energies shown in Fig. 5 and a mean particle content of the midrapidity source of 50%, we display in Fig. 7 calculated pseudorapidity distribution functions for charged particles at LHC energies for successive variations of the transport parameters.

In the upper (first) frame results for three different values of the time parameter  $x=\tau_{int}/\tau_y$  are shown. In the second frame the dependence of the distribution function on the widths of the peripheral distributions is displayed. The third frame gives the dependence on the width of the midrapidity source, and the fourth frame the dependence on the particle content of the midrapidity source. In each case, the values of those parameters that are not varied are taken from the extrapolation in Fig. 5 with the time parameter from (15). Considerable changes in the shapes of the distribution functions are observed, in particular, for varying particle content of the central source. It is presently not possible to predict the particle content of the midrapidity source at LHC energies on the basis of the two Au + Au results at 130 GeV and 200 GeV.

Our diffusion-model result with extrapolated parameters from Fig. 5 and 50% particle content in the central source is shown in Fig. 8, curve [A]. For comparison, the RHIC result for  $\sqrt{s_{NN}}=200$  GeV of Fig. 3 is redisplayed in the upper frame together with this prediction. It is significantly narrower than the LHC result. Here the time parameter is taken from (15). Curve [B] in the lower frame is obtained with the exponential extrapolation of the time parameter as shown in Fig. 5. This yields a smaller value of  $x$  and hence, a broader distribution function. Values of the particle content of the midrapidity source, and of the widths are given in the caption.

We have also obtained diffusion-model results using the extrapolations of the number of produced particles at midrapidity given by other authors. In particular, curve [C] in Fig. 8 uses a logarithmic extrapolation of the midrapidity value that yields  $dN/d\eta \simeq 1100$  [18], whereas the saturation model [18] predicts  $dN/d\eta \simeq 1800$  at midrapidity, with the resulting diffusion-model distribution [D]. The forthcoming LHC data are likely to be in between the limiting cases [A] and [D].

Whereas the precise midrapidity value will have to be determined from the data, it is the detailed shape of the distribution function that will be of interest in order to determine to what extent the system reaches statistical equilibrium at LHC energies.

It is interesting to compare our results with predictions of other models that are not

based on nonequilibrium-statistical mechanics, but instead on QCD. In particular, there are calculations within the framework of the Parton Saturation Model that not only predict the midrapidity value, but also the full distribution function at RHIC energies, and also at LHC [19]. These calculations are based on a classical effective theory that describes the gluon distribution in large nuclei at high energies where saturation might occur at a critical momentum scale, to form a so-called Color Glass Condensate (CGC) [20].

Although this assumption has a clear and reasonable physical basis, the resulting pseudorapidity distribution functions for produced charged hadrons at the available RHIC energies do not appear to match the precision of our phenomenological three-sources diffusion-model results when compared to the existing data. At LHC energies, the overall CGC-distribution for a given midrapidity value as obtained with the assumption of a constant  $\alpha_s$  for strong coupling is slightly narrower than the corresponding diffusion-model result.

Additional consideration of a running coupling gives a midrapidity value that is of the order of 10% smaller; another uncertainty arises from the extrapolation of the saturation scale to LHC energies. Various predictions for central rapidity densities and pseudorapidity distributions at RHIC and LHC energies had been summarized e.g. in [21], where also the differences among the existing models - including hydrodynamical and pQCD approaches and their numerical implementations - had been discussed. It appears, however, that the analytical diffusion-model approach provides better results when compared in detail to the experimental distribution functions at RHIC energies.

## 5 Conclusion

We have described net-proton and charged-hadron distributions in collisions of heavy systems at SPS and RHIC energies in a Relativistic Diffusion Model (RDM) for multiparticle interactions. Analytical results for the rapidity distribution of net protons in central collisions are found to be in good agreement with the available data. An extrapolation of the rapidity distributions for net protons to LHC energies has been performed. The precise number of particles in the midrapidity source remains uncertain at LHC energies and will have to be determined from experiment.

At RHIC - and most likely at LHC -, a significant fraction of the net protons (about 14 per cent at 200 GeV for Au + Au) reaches local statistical equilibrium in a fast and discontinuous transition which we believe to indicate parton deconfinement.

In the three-sources RDM, we have calculated pseudorapidity distributions of produced charged hadrons. The diffusion-model parameters have been optimized in a  $\chi^2$ -minimization with respect to the available PHOBOS data in Au+Au and Cu+Cu [22] at RHIC. Excellent results for the energy dependence of the distribution functions have been obtained.

In an extrapolation to LHC energies, the pseudorapidity distribution for produced charged hadrons has been calculated. Here the essential parameters relaxation time, diffusion coefficients or widths of the distribution functions of the three sources, and number of particles in the local equilibrium source will have to be adjusted once the ALICE data for Pb + Pb have become available in 2009.

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**Table 1.** Produced charged hadrons for central (0-6%) collisions of Cu + Cu at RHIC energies of  $\sqrt{s_{NN}} = 62.4$  GeV and 200 GeV, of Au + Au at 19.6 GeV, 130 GeV, and 200 GeV, and of Pb + Pb at LHC energies in the Relativistic Diffusion Model. The number of produced charged particles is  $N_{ch}^{tot}$  with  $N_{ch}^{1,2}$  for the sources 1 and 2 and  $N_{ch}^{eq}$  for the equilibrium source, the percentage of charged particles produced in the locally thermalized source is  $n_{ch}^{eq}$ . The ratio  $\tau_{int}/\tau_y$  determines how fast the system of produced charged particles equilibrates in rapidity space. The effective widths of the peripheral sources are  $\Gamma_{1,2}^{eff}$ , of the midrapidity source  $\Gamma_{eq}^{eff}$ . The  $\chi^2/d.o.f.$  is as shown in Figs. 3, 4, with  $d.o.f.$  = number of data points - number of free parameters.

system and energy	$N_{ch}^{tot}$	$N_{ch}^{1,2}$	$N_{ch}^{eq}$	$n_{ch}^{eq}(\%)$	$\frac{\tau_{int}}{\tau_y}$	$\Gamma_{1,2}^{eff}$	$\Gamma_{eq}^{eff}$	$\frac{\chi^2}{d.o.f.}$
Cu+Cu 62.4 GeV	825	400	26	3.2	1.12	3.70	5.15	$\frac{4.7}{49}$
Cu+Cu 200 GeV	1474	685	105	7.1	1.08	4.03	2.45	$\frac{2.0}{49}$
Au+Au 19.6 GeV	1692	846	-	-	1.23	2.90	-	$\frac{0.7}{28}$
Au+Au 130 GeV	4233	1837	560	13.2	1.02	3.56	2.64	$\frac{3.7}{49}$
Au+Au 200 GeV	5123	1887	1349	26.3	0.93	3.51	3.20	$\frac{1.1}{49}$
Pb+Pb 5520 GeV	9520	2380	4760	50	0.91	4.6	7.5	-

## Figure captions

**Fig. 1.** Dependence of the Diffusion-Model parameters for heavy systems (Au + Au at AGS and RHIC, Pb + Pb at SPS and LHC) on the center-of-mass energy  $\sqrt{s_{NN}}$ , with extrapolations to LHC (top to bottom frame): Quotient of interaction time and relaxation time - see text for dashed curve; rapidity width coefficient including collective expansion; and longitudinal collective velocity. The results are for net-proton rapidity distributions.

**FIG. 2.** Net-proton rapidity spectra in the Relativistic Diffusion Model (RDM), solid curves: Transition from the double-humped shape at SPS energies of  $\sqrt{s_{NN}} = 17.3$  GeV to a broad midrapidity valley in the three-sources model at RHIC (200 GeV) and LHC (5.52 TeV). The two curves at LHC energies contain 14% and 7% of the net protons in the midrapidity source, respectively. Kinematic constraints will modify the LHC-distributions at large values of  $y$ . Data are from NA49 at SPS [9], and BRAHMS at RHIC [2].

**FIG. 3.** Calculated pseudorapidity distributions of produced charged hadrons from central Au + Au collisions at  $\sqrt{s_{NN}} = 19.6$  GeV, 130 GeV and 200 GeV. The analytical RDM-solutions (two sources at the lowest, three at the higher energies) are optimized in a  $\chi^2$ -fit with respect to PHOBOS data [3]. The percentage of particles in the midrapidity source is indicated.

**FIG. 4.** Calculated pseudorapidity distributions of produced charged hadrons from central Cu + Cu collisions at  $\sqrt{s_{NN}} = 62.4$  GeV and 200 GeV. The analytical RDM-solutions for three sources are optimized in a  $\chi^2$ -fit with respect to PHOBOS data [23]. The percentage of particles in the midrapidity source is indicated.

**Fig. 5.** Dependence of the Diffusion-Model parameters for heavy systems (Au + Au at AGS and RHIC, Pb + Pb at LHC) on the center-of-mass energy  $\sqrt{s_{NN}}$ , with extrapolations to LHC (arrows): Quotient of interaction time and relaxation time for sinh- and exponential extrapolation (upper frame); rapidity width of the peripheral



sources including collective expansion (middle frame); width of the midrapidity source (lower frame). The results are for charged-hadron rapidity distributions.

**FIG. 6.** Total number of produced charged hadrons per number of participant nucleons as function of the center-of-mass energy as obtained from the RDM-results for d+Au, Cu+Cu, Au+Au, and for Pb+Pb [9] pseudorapidity distributions. The extrapolation to LHC energies used in this work is also shown.

**FIG. 7.** Pseudorapidity distributions of produced charged hadrons for central Pb + Pb collisions at LHC energies of 5520 GeV with a charged-particle content in the central source of 50%, extrapolated transport parameters from Fig. 5, and total number of produced charged hadrons from Fig. 6. Upper (first) frame: Results for three different values of the time parameter  $x = \tau_{int}/\tau_y$ . Second frame: Dependence of the distribution function on the widths of the peripheral distributions. The result for  $\Gamma_{1,2}=4.6$  is for the extrapolated values of all parameters as in curve [A] of Fig. 8. Third frame: Dependence on the width of the midrapidity source. Fourth frame: Dependence on the particle content of the midrapidity source. The values of those parameters that are not varied are taken from the extrapolation in Fig. 5.

**FIG. 8.** Calculated pseudorapidity distributions of produced charged hadrons for central Au + Au collisions at RHIC compared with 200 A GeV PHOBOS data [3], and diffusion-model extrapolation to Pb + Pb at LHC energies of 5520 GeV with a charged-particle content in the central source of 50%, transport parameters from Fig. 5, and total number of produced charged hadrons from Fig. 6 (upper frame). Lower Frame: Curve [A] is the diffusion-model result as in the upper frame. Curve [B] is obtained with the exponential extrapolation of the time parameter shown in Fig. 5 and  $\Gamma_{1,2} = 5.9$  (cf. Fig. 7); here the particle content of the midrapidity source is 40%. Curve [C] uses a logarithmic extrapolation of the midrapidity value that yields 1100 [18], with  $\Gamma_{1,2} = 5.9$ . The saturation model [18] predicts  $dN/d\eta \simeq 1800$  at midrapidity with the resulting distribution [D] using  $\Gamma_{1,2} = 5.9$ .

















